The Zero of Potential Energy Is Not Arbitrary

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Matter & Interactions I: Modern Mechanics

First semester of an introductory calculus-based physics curriculum

Start with $\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$

It follows that $E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$

Two interacting particles

(low speed so U is meaningful)

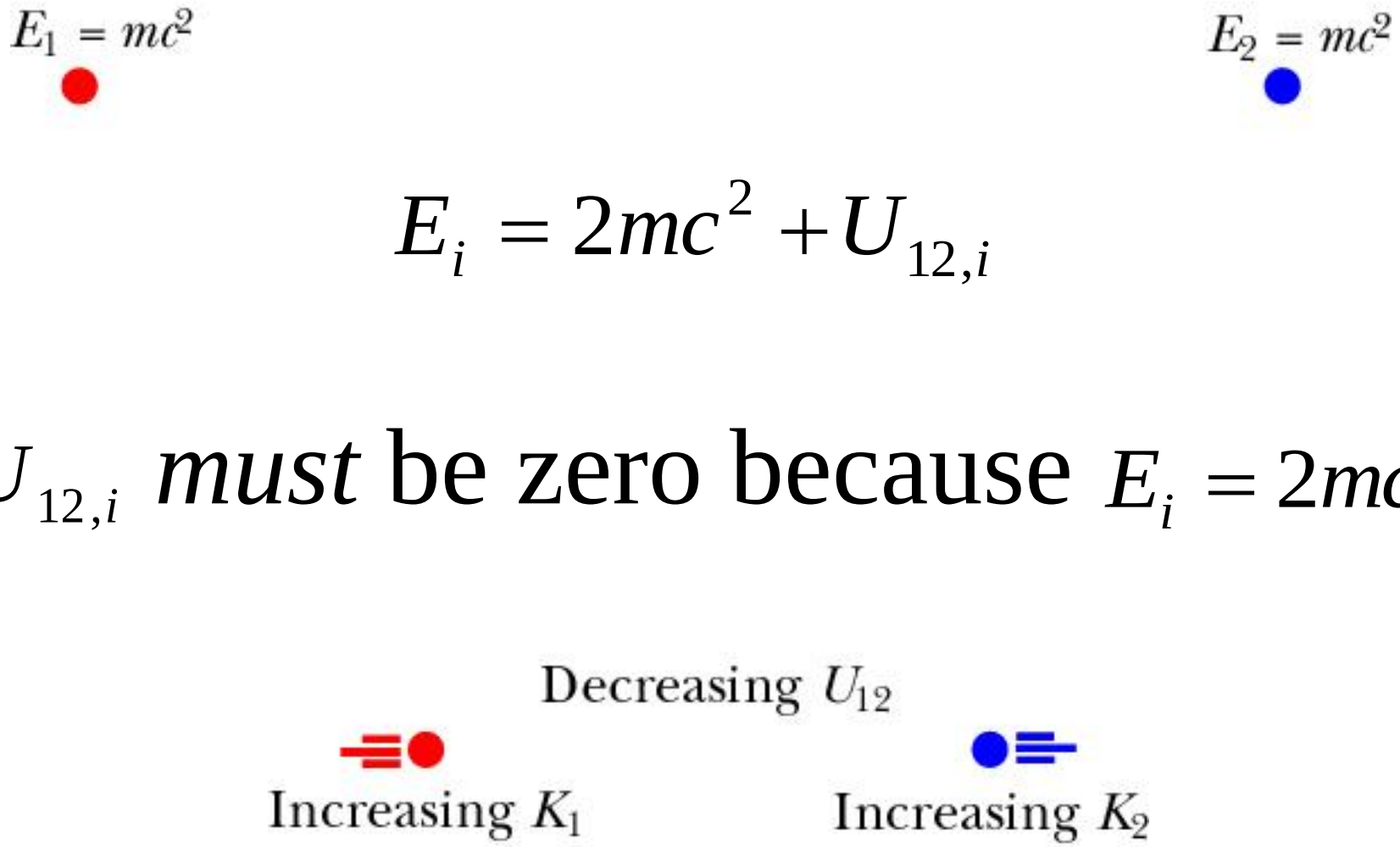
$$\Delta\left[\frac{m_1c^2}{\sqrt{1-v_1^2/c^2}}+\frac{m_2c^2}{\sqrt{1-v_2^2/c^2}}\right]+\Delta U_{12}=W+Q$$

$$\Delta\left(m_1c^2+m_2c^2+K_1+K_2\right)+\Delta U_{12}=W+Q$$

Modern Mechanics homework problem

Positron and electron released from rest very far from each other.

(a) Graph the various energies involved in this process, as a function of separation...


$E_1 = mc^2$
$E_2 = mc^2$
$E_i = 2mc^2 + U_{12,i}$
$U_{12,i}$ *must* be zero because $E_i = 2mc^2$
Decreasing U_{12}
Increasing K_1
Increasing K_2
$E_f = 2mc^2 + K_1 + K_2 + U_{12,f} = E_i$

$$E = 2mc^2$$

If we add an arbitrary constant to the energy of this system, the energy and momentum of the system won't transform properly to another reference frame.

Another way of seeing this is that

$$E^2 - (pc)^2 = \left(M_{\text{total}} c^2\right)^2 = \left(2mc^2\right)^2$$

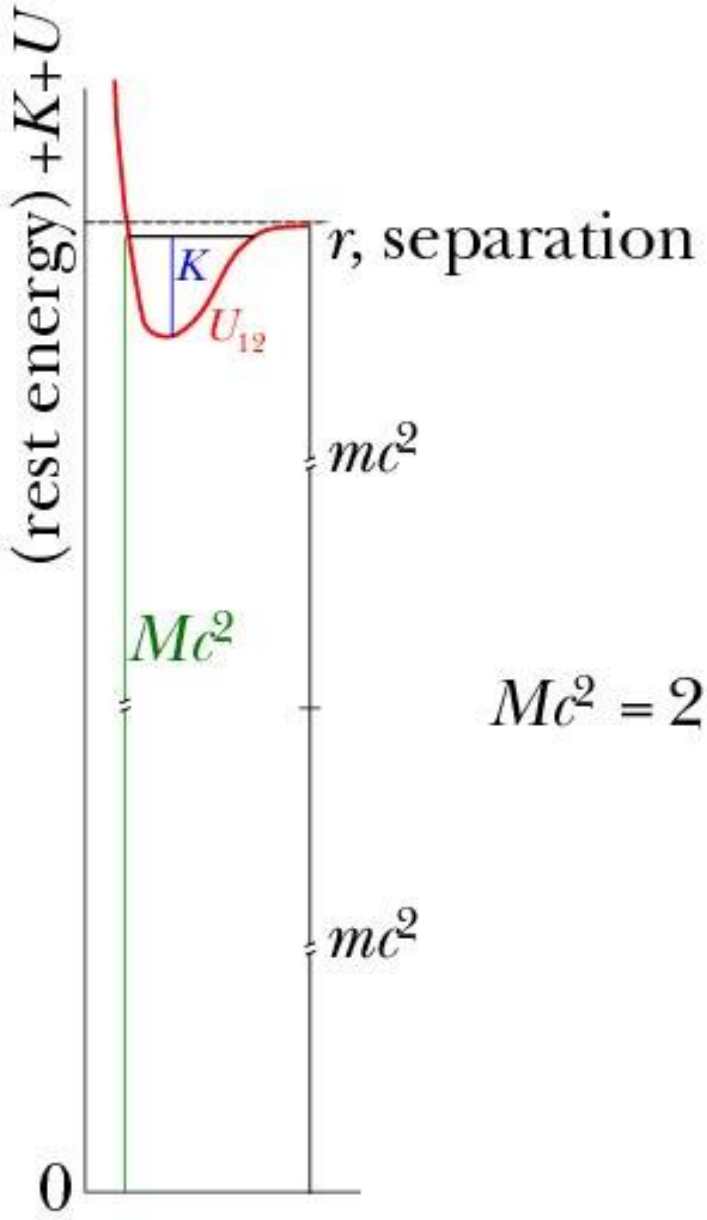
must be an invariant. Adding an arbitrary constant to E would destroy the invariance.

Pedagogical consequences

- Relativistically correct
- Total energy never negative, so less discomfort with negative U
- Students more comfortable with absolute than relative quantities

Oxygen molecule

(*U* greatly exaggerated)



$$Mc^2 = 2mc^2 + K + U_{12}$$

Fred Reif points out that this is similar to the situation with entropy. Pre-quantum it had an arbitrary additive constant. Post-quantum, entropy has an absolute value.

Acknowledgements

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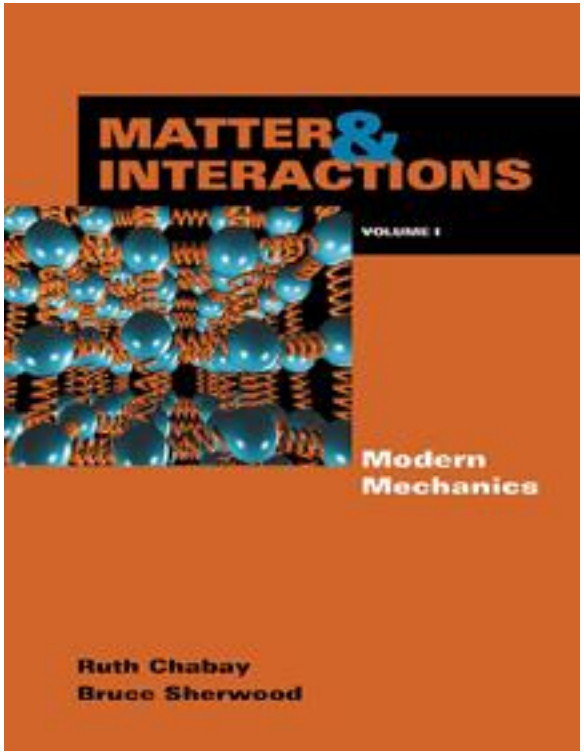

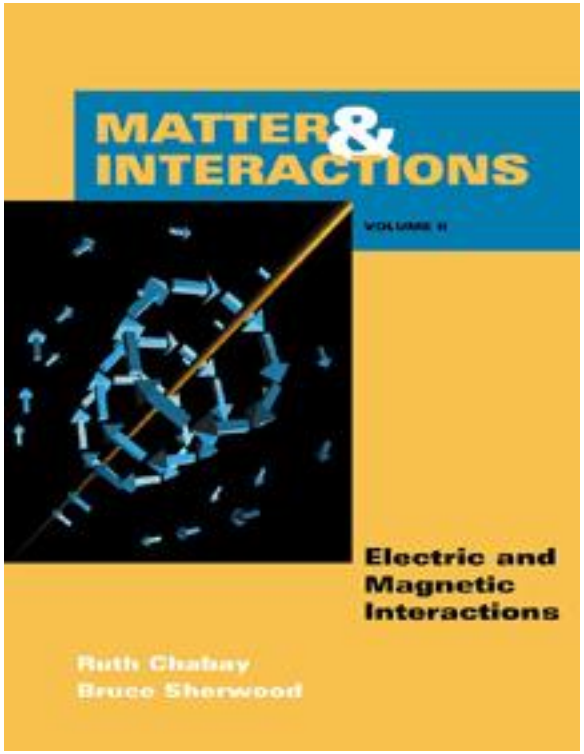

John Wiley & Sons, 2002

http://www4.ncsu.edu/~rwchabay/mi