ACKNOWLEDGMENT

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[1]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[2]B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[3]P. W. Anderson, in *Lectures on the Many-Body Problem*, edited by E. R. Caianello (Academic, New York, 1964), Vol. II, p. 113.
[4]Richard P. Feynman, Robert B. Leighton, and Matthew Sands, *Lectures on Physics* (Addison-Wesley, New York, 1965), Vol. III, Chap. 21, p. 14.
[5]P. W. Anderson, Phys. Rev. **112**, 1900 (1958).
[6]Patrick A. Lee and Marlan O. Scully, Phys. Rev. B **3**, 769 (1971).
[7]Leon N. Cooper, Phys. Rev. **104**, 1189 (1956).
[8]D. J. Thouless, Phys. Rev. **117**, 1256 (1960).
[9]Andrew L. DiRienzo and Richard A. Young, Phys. Rev. B **25**, 6684 (1982). For additional information on charged superconductors see: R. E. Glover III, S. K. Ghosh, and W. E. Daniels, Jr., in *Proc. Int. Symposium on Basic Problems in Thin Film Physics*, Clausthal, September 1965, edited by R. Niedermeyer and H. Mayer (Vandenhoeck and Reprecht, Göttingen, Germany, 1966), p. 536; A. Bardasis, Phys. Rev. B **26**, 1477 (1982).
[10]M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Lett. **8**, 316 (1962).
[11]P. R. Wallace and M. J. Stavn, Can. J. Phys. **43**, 411 (1965).

Pseudowork and real work

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In teaching mechanics, we should more clearly distinguish between an integral of Newton's second law and the energy equation. This leads to greater clarity in the notions of system, work, and energy. A reorientation of the treatment of work and energy would not only provide benefits in the mechanics course but would also produce better connections between the mechanics and thermodynamics courses.

I. SOME PUZZLES

When a car accelerates from rest, it appears that the kinetic energy is equal to the work done by the frictional force exerted by the road, acting through the displacement of the car. Yet the frictional force does *no* work, and the car's kinetic energy comes from the burning of gasoline, not from the road.

When a block slides down an incline with friction, it is often said that the kinetic energy is equal to the work done by gravity minus the work done by the frictional force. Yet one knows that the block gets warmer, and there is the uneasy feeling that this increase in thermal energy ought to appear explicitly in the work–energy equation.

These mechanics problems are representative of a general class of situations involving systems which cannot be treated as point particles. The uneasiness provoked by such problems is only partially offset by the fact that one often seems to get the "right" answer. The goal of this article is to resolve the ambiguities and deepen our understanding. A new approach not only can clarify such situations but can also improve the connections between the introductory mechanics course and the following thermodynamics course.

II. PSEUDOWORK

Erlichson[1] and Penchina[2] have pointed out that we frequently do not properly distinguish between the work–energy equation of mechanics and a particular integral of Newton's second law. Take the second law for a system of particles,

$$\sum \mathbf{F}_{i_{\text{external}}} = M\mathbf{a}_{\text{CM}},$$

and integrate through a displacement of the center-of-mass point (interchanging summation and integration):

$$\int \Big(\sum \mathbf{F}_{i_{\text{external}}}\Big) \cdot d\mathbf{r}_{\text{CM}} = \int M \frac{d\mathbf{v}_{\text{CM}}}{dt} \cdot d\mathbf{r}_{\text{CM}};$$

$$\sum \Big(\int \mathbf{F}_{i_{\text{external}}} \cdot d\mathbf{r}_{\text{CM}}\Big) = \Delta\,(\tfrac{1}{2} Mv_{\text{CM}}^2).$$

Penchina calls the term on the left-hand side the total "pseudowork." It is *not* equal to the total real work done on the system, because the forces have been multiplied by the center-of-mass displacement rather than by their individual displacements. The right-hand side of the equation is *not* in general equal to the kinetic energy change of the system, since it involves only the center-of-mass speed. The term $\frac{1}{2} Mv_{\text{CM}}^2$ is sometimes called "the kinetic energy of the center of mass." Although "pseudowork–energy equation" is a good name for this relationship, it has been found useful for teaching purposes to call it the CM (center-of-mass) equation. This name emphasizes that the equation does not really deal with work and energy but is associated with center-of-mass quantities.

The displacement $d\mathbf{r}_i$ of the point of application of the ith force is not necessarily equal to the displacement $d\mathbf{r}_{\text{CM}}$

of the center-of-mass point, so that for the ith force

$$\int \mathbf{F}_{i_{\text{external}}} \cdot d\mathbf{r}_{\text{CM}} \neq \int \mathbf{F}_{i_{\text{external}}} \cdot d\mathbf{r}_i;$$

$$\text{pseudowork}_i \neq \text{work}_i.$$

For point particles $d\mathbf{r}_i = d\mathbf{r}_{\text{CM}}$, the pseudowork equals the work, and the energy of a point particle is purely kinetic, so that the CM and work–energy equations are equivalent.

If one starts from the x component of the equation of motion, one finds

$$\sum\left(\int F_{x,i_{\text{external}}}\, dx_{\text{CM}}\right) = \Delta\,(\tfrac{1}{2}\, M v_{x,\text{CM}}^2),$$

with similar equations for y and z. The *algebraic* sum of these three equations yields the full CM equation.

III. ILLUSTRATIVE EXAMPLE

The differences between the CM equation and the work–energy (WE) equation can be appreciated by writing both equations for a variety of situations. Consider a cylinder rolling without slipping a distance d_{CM} down an incline (Fig. 1). Take the cylinder as the system, so that Mg is an *external* force applied by the Earth to the cylinder:

$$\text{CM, cylinder:}\quad (Mg \sin\theta - f)d_{\text{CM}} = \Delta\,(\tfrac{1}{2}\, \text{M} v_{\text{CM}}^2),$$

$$\text{WE, cylinder:}\quad (Mg \sin\theta)d_{\text{CM}} = \Delta\,(\tfrac{1}{2}\, M v_{\text{CM}}^2) + \Delta\,(\tfrac{1}{2}\, I\omega^2).$$

Both of these equations are correct. Each gives different kinds of information, so one or the other may be more useful in determining certain aspects of the motion. Notice that the frictional force appears in the CM equation, since it contributes to the resultant force ($Mg \sin\theta - f$), but it does not appear in the work–energy equation. The point of application of the frictional force is at the rolling contact point, which is always instantaneously at rest. Hence the frictional force acts through zero distance and does no (real) work, though it can be said to perform an amount $-fd_{\text{CM}}$ of pseudowork.

Also note that the rotational energy term is missing from the CM equation, since the CM equation, despite appearances, is not really an energy equation. The CM equation is merely the spatial integral of

$$\sum \mathbf{F}_{i_{\text{external}}} = M\mathbf{a}_{\text{CM}}$$

and deals not with energy but with the motion of the (mathematical) center-of-mass point. If each force acts on a system through a displacement equal to the center-of-mass displacement, then the CM and WE equations are the same. One example of such a system is a point particle. However, the frictional force and the gravitational force act on the rolling cylinder through different distances, and this leads to differences between the CM and WE equations. Such differences can arise for deformable systems and also for rotating rigid systems.

If we include the Earth in our chosen system, the Mg force is no longer an external force and the work–energy equation becomes

$$\text{WE, universe:}\quad 0 = \Delta\,(\tfrac{1}{2}\, M v_{\text{CM}}^2) + \Delta\,(\tfrac{1}{2}\, I\omega^2) - (Mg \sin\theta)d_{\text{CM}}.$$

The Mgh term appears on the right-hand side as (negative) change in the gravitational potential energy when we choose the universe as the system of interest, but it appeared on the left-hand side as positive work when we chose the cylinder as the system. We will see later that it is important to be very clear about the choice of system, since it is *external* forces that perform the external work which appears on the left-hand side of the work–energy equation. As Penchina says, application of both the CM and WE equations helps students better understand and appreciate work and energy. Similarly, writing the WE equation for more than one choice of system illuminates the difference between work as a process and energy as a change of state.

IV. ENERGY EQUATION

Next consider an accelerating car. Let it be an electric car to avoid questions of air intake and of exhaust. Take the car as the system (Fig. 2), and ignore air resistance. The forces f_1 and f_2 represent the total forces on the rear and front wheels. These forces *do no work,* since the point of application of these forces has no displacement if the wheels do not slip. The CM equation predicts how these forces change the center-of-mass velocity:

$$\text{CM:}\quad (f_1 + f_2)d_{\text{CM}} = \Delta\,(\tfrac{1}{2}\, M v_{\text{CM}}^2).$$

This CM equation is precisely equivalent to $f_1 + f_2 = Ma_{\text{CM}}$, from which it was derived. The frictional forces make possible an acceleration of the center of mass, but they do no work, and the left-hand side of the CM equation is *not* the work done on the car. Similarly, the right-hand side of this CM equation contains not the energy of the car but only the "kinetic energy of the center of mass." The full energy of the car includes several other terms. The energy equation for the car is the following (neglecting air resistance):

$$Q_{\text{net}} = \Delta\,(\tfrac{1}{2}\, M v_{\text{CM}}^2) + \Delta\,\text{KE}_{\text{internal}} + \Delta E_{\text{thermal}} + \Delta E_{\text{battery}}.$$

Q_{net} is the net heat transfer into the car from the surroundings, consisting mainly of (negative) heat transfer from the hot engine to the air and from the hot tires to the cooler pavement. $\Delta\,\text{KE}_{\text{internal}}$ represents the increased energy of motion of the internal parts of the car, including the engine and the wheels. $\Delta E_{\text{thermal}}$ is associated with the temperature rise of the engine and battery (friction, ohmic heating, and irreversible aspects of battery discharge). $\Delta E_{\text{battery}}$ is the (negative) change in chemical energy which pays for all the other terms in the equation.

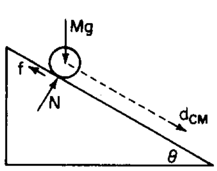

Fig. 1. Cylinder rolls without slipping down an incline.

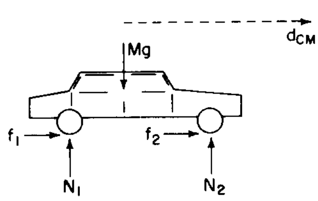

Fig. 2. Car accelerates with the wheels not slipping. Air resistance is neglected.

This energy equation comes from the general form

net external inputs to a system (mechanical work, heat transfer, mass transfer, radiation, etc.)	=	change in energy of a system (kinetic energy, gravitational potential energy, chemical energy, etc.)

For historical reasons this is called the first law of thermodynamics, although the energy principle is pervasive throughout science and is not really tied to thermodynamics or to thermodynamics courses. It cannot of course be derived from Newton's laws, although it contains the purely mechanical work–energy (WE) equation as a subset of the full possibilities. It would be natural to call it simply "the energy equation." However, at present the label "work–energy equation" is often applied indiscriminately in mechanics courses to the very different CM and WE equations. For that reason it seems prudent to drop the term "work–energy equation" entirely, and use two new terms in the mechanics course: the CM equation and the FLT (first law of thermodynamics). This terminology has the additional advantage that the energy principle is known as the first law in the thermodynamics course which normally follows the mechanics course. At some future date, if the terminology "work–energy equation" is no longer used, it would be sensible to rename the FLT simply "the energy equation," both in the mechanics course *and* in the thermodynamics course. It should not be called the work–energy equation, because work is only one of many processes which can carry energy across the system boundary.

(There is a minor notational problem in the transition from mechanics to thermodynamics. In mechanics it is natural to call work done *on* the system positive, writing $W = \Delta E$, whereas in thermodynamics we usually define work done *by* the system as positive, due to the emphasis on deriving useful work output from heat transferred into the system, with $Q = \Delta U + W$. This notational difference should be pointed out to the student in the thermodynamics course.)

V. ADDITIONAL EXAMPLES

Because the distinction between the CM equation and the first law of thermodynamics has usually not been made in standard textbooks, there is a shortage of relevant homework and exam problems in this area. Some useful problems have been invented for a computer-based mechanics course[3–6] which attempts to treat the energy topic comprehensively. These problems will be summarized here both to further illustrate the principles and to stimulate others to develop additional helpful problems.

The common theme of these problems is a change of shape of the chosen system, which may consist of several bodies. This may involve, for example, an unwinding chain or a mountain climber scaling a cliff (where there is a growing separation of climber and Earth in a system containing both). We begin with three examples where there is little or no configurational energy associated with the deformation of the system.

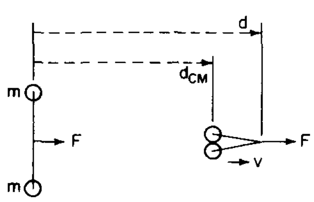

Fig. 3. Two pucks on a frictionless table accelerate from rest. The point of application of the force goes farther than the center-of-mass point.

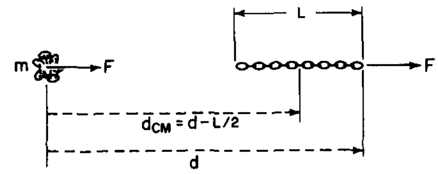

Fig. 4. Chain at rest in a small heap on a frictionless floor is opened out by pulling with a constant force. The force moves farther than the center of mass does.

An exam problem by Michael Weissman involved two pucks tied together and lying at rest on a frictionless table. The center of the string is pulled with a constant force perpendicular to the string as shown in Fig. 3, and the pucks collide inelastically. When the stuck-together pucks have attained a speed v, we have the following equations (where FLT stands for the first law of thermodynamics):

$$\text{CM:}\quad Fd_{\text{CM}} = \tfrac{1}{2}(2m)v^2,$$

$$\text{FLT:}\quad Fd = \tfrac{1}{2}(2m)v^2 + \Delta E.$$

The external force acts through a distance d greater than d_{CM}, the distance the center of mass moves. The term ΔE is the increase in internal energy, corresponding to the mechanical energy lost to other forms during the inelastic collision. (If there is significant radiation of sound or heat, or thermal conduction to the table, these transfers should appear as negative quantities on the left-hand side of the FLT, though here they have been lumped into the term ΔE.) Typically, the student is asked to find v and ΔE, given F, d, and the lengths of the strings.

A related problem involves uncoiling a uniform chain of length L made of metal links. The chain initially is bunched up in a small heap (Fig. 4). A constant force pulls the chain across a frictionless floor. The chain reaches a speed v after opening out completely:

$$\text{CM:}\quad F(d - L/2) = \tfrac{1}{2}mv^2,$$

$$\text{FLT:}\quad Fd = \tfrac{1}{2}mv^2 + \Delta E.$$

Again, ΔE is the increase in internal energy of the chain (neglecting energy transfers out of the system) and can be thought of mainly as related to an increase in temperature of the chain after inelastic collisions among the links of the chain have damped down. From the CM and FLT equations the student may be asked to determine v and ΔE.

For a third problem, consider two identical blocks initially at rest on a frictionless table, pulled away from each other by equal forces (Fig. 5). Take the *two* blocks as the system of interest:

$$\text{CM:}\quad (f - f)\Delta x_{\text{CM}} = \tfrac{1}{2}(2m)v_{\text{CM}}^2 = 0,$$

$$\text{FLT:}\quad 2fd = 2(\tfrac{1}{2}mv^2);$$

v_{CM} remains zero, and the center of mass does not move. Each block does acquire a speed v. Clearly, work is done on

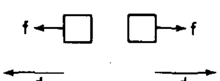

Fig. 5. System composed of two blocks at rest is pulled apart by equal and opposite forces. The center of mass does not move.

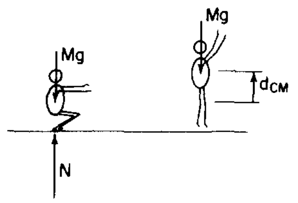

Fig. 6. Jumper leaps up from a crouching rest position. The normal force exerted upwards by the floor does no work, since there is no displacement of the contact point.

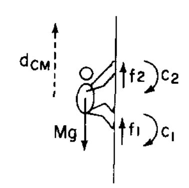

Fig. 7. Climber moves slowly up a vertical cliff. The cliff applies forces and couples to the hands and feet but does no work, since there is no displacement at the contact points. The Earth does negative work on the climber.

the system of the two blocks, but there is no change in v_{CM}^2. This problem can be used to illustrate to the student in a very simple situation the major differences between the CM equation and the FLT. Similar problems are the symmetric stretching of a spring, or a weightlifter slowly raising a barbell (the Earth and barbell being the chosen system). In each of these cases the net force is zero, so the center of mass does not accelerate, but the individual forces do work. Consideration of such processes can lead in a natural way to the introduction of the concept of potential energy.

Next we consider systems where configurational energy is important. A rich class of such examples involves men and women jumping or climbing. If there is no slippage between foot or hand and the supporting surface, the support forces though large do no external work on the person. Consider a jumper who leaps straight up from a crouching position, with the center of mass rising d_{CM} at the moment of liftoff (Fig. 6). $\overline{N}$ is the average value of the force of the floor exerted upward on the jumper's foot during contact. Nelgect heat transfer to the air.

$$\text{CM:}\quad (\overline{N} - Mg)d_{CM} = \tfrac{1}{2} Mv_{CM}^2,$$

$$\text{FLT:}\quad -Mgd_{CM}$$

$$= \tfrac{1}{2} Mv_{CM}^2 + \Delta\ \text{KE}_{\text{internal}} + \Delta E_{\text{thermal}} + \Delta E_{\text{chemical}}.$$

Because the CM equation involves $\overline{N}$, it can be used to estimate the required strength of the floor, about which the FLT says nothing. $\Delta\ \text{KE}_{\text{internal}}$ includes flailing of arms and legs. $\Delta E_{\text{thermal}}$ represents the temperature rise of the body prior to heat transfer to the surrounding air. $\Delta E_{\text{chemical}}$ is negative and represents payment for the other terms. If we take the universe as the system, the Mgd_{CM} term appears on the right-hand side of the FLT equation as increased gravitational potential energy due to increased separation of jumper and Earth. In that case, no work is done on the system. The student can be given d_{CM} and the final height to which the jumper rises, from which can be calculated the average normal force exerted by the floor. The FLT can be used to calculate the change in the internal energies of the jumper, which gives a minimum value for the chemical energy expended.

A climber inches *slowly* up a vertical cliff. The cliff exerts upward forces and also applies couples (Fig. 7). Taking the climber as the system of interest we have

$$\text{CM:}\quad (f_1 + f_2 - Mg)d_{CM} = 0,$$

$$\text{FLT:}\quad -Mgd_{CM} - Q_{\text{loss}} = \Delta E_{\text{chemical}}.$$

The forces exerted by the cliff do no work, since without slippage the point of contact does not move. The climber burns chemical energy to compensate for the negative work done by the external Mg force and for the heat transfer Q_{loss} to the air. Again, if we take the universe as the system the Mgd_{CM} term appears on the right-hand side as change in gravitational potential energy, and the Q_{loss} term appears on the right-hand side as increased thermal energy in the atmosphere. The student can be asked about the sum of the chemical energy and heat transfer terms.

One frequently hears statements about such a situation that "the climber does work to increase his gravitational potential energy." There is *no* system on which the climber does work, since the cliff and the Earth do not move. Also, one can properly speak of "gravitational potential energy" not of the climber alone but only in terms of separation of climber and Earth in the larger system. It is unfortunately very common to speak of the gravitational potential energy of a person or of a falling rock, but such statements really should be avoided, since the Earth must be included in the system. Lack of clarity on this point leads students to make the mistake of putting the mgh in twice when analyzing a falling rock: $mgh = \Delta\ \text{KE} - mgh$. Here the student has mixed two different choices of system, the rock alone (on which the Earth does external work mgh) and the universe (in which there is a change of gravitational potential energy $-mgh$). Avoiding this kind of double-entry mistake is helped by analyzing a problem with two or more different choices of systems. It is just as important in work–energy problems as it is in force–acceleration problems to be clear about the choice of system. Emphasizing choice of system and the use of freebody diagrams in the work–energy part of the mechanics course provides another opportunity to practice these important concepts.

Similar remarks apply to the way in which PE $= mgy$ is derived. One should be very careful to distinguish between the two choices of system (rock alone, or rock plus Earth). Indeed, it is the comparison of the energy relations for these two systems from which one can determine the potential energy change.

A sprinter accelerating or a woman walking upstairs involves contact forces on the feet which do no work on the person. In some ways foot locomotion is a better introduction to forces which accelerate but do no work than is the car example, since the motionlessness of the bottom of a nonslipping wheel is much harder to grasp than the nonslip action of the walking or running foot.

Some recent textbooks have brief sections dealing with the CM equation. For example, a 1981 revision of the popular textbook by Halliday and Resnick[7] includes a new section (pp. 137–140) on this topic. The examples in the text and the homework problems include a woman jumping upwards, a car decelerating without slipping, an astronaut pushing away from a spaceship, and an ice skater using his hands to stop at a wall. These are all examples of a contact force acting through zero distance.

One might protest that the distinctions brought out by such problems are overly pedantic. This is partly a matter of taste. But generations of physics teachers have been careful to explain to students the precise technical meaning of the word "work," and that holding a book at arm's length may be tiring but you are doing no work on the book. If we can be careful about this trivial case, we should be even more careful, not less, when discussing the more complicated cars and climbers that show up throughout the mechanics course.

Moreover, if these distinctions are not made one obtains unphysical results when frictional forces do work. As will be shown in a forthcoming article,[8] frictional forces generally act through a distance which is less than the displacement of the center of mass. In rolling without slipping, the frictional force acts through zero distance and does no work. For a block sliding on a table, deformation of the rubbing surface causes the effective displacement d_{eff} of the frictional force to be less than the center-of-mass displacement d_{CM}. The resulting difference between the pseudowork $-\mu N d_{\mathrm{CM}}$ and the real work $-\mu N d_{\mathrm{eff}}$ is numerically equal to the rise in the thermal energy of the block.

VI. PEDAGOGICAL AND PHILOSOPHICAL ISSUES

It has been standard practice not to mention the first law of thermodynamics in mechanics courses. The work–energy equation is treated on a purely mechanical level, with considerable formal attention paid to external and internal forces, to conservative and nonconservative forces, and to mechanical potential energy. This approach has serious drawbacks. It is overly formal for an introductory college mechanics course, it does not permit the application of energy relationships to very common problems usually treated in mechanics courses, and it produces an artificial and unnecessary separation between mechanics and thermodynamics. The main justification for this practice is that it leads to treating both mechanics and thermodynamics as self-contained axiomatic systems, thus providing practice in manipulating formal structures and reflecting the historical development of both subjects. However, we have seen that Newtonian mechanics alone is not in fact a consistent system for handling the energy aspects of many situations which have traditionally been treated in the mechanics course.

It is certainly possible to restrict the mechanics course solely to Newtonian mechanics, but in that case there are two reforms that should be made. First, the CM and work–energy equations should be carefully distinguished from each other, since otherwise our careful definitions of work and energy are compromised. Second, those situations whose energy relationships involve more than Newtonian mechanics should be removed from the course: accelerating cars, inelastic collisions, people climbing or jumping, blocks sliding down inclines with friction, etc. This would be a very heavy price to pay, yet it is a price which must be paid to keep the axiomatic approach at least consistent within itself.

Given the stringent limitations of a purely Newtonian mechanics course, it seems far more important to teach *physics* than to restrict ourselves to teaching rational *mechanics*. We often mention special relativity in a mechanics course, even though Einstein lived long after Newton. We should not hesitate to introduce the first law of thermodynamics when it is needed, even though this energy principle emerged in the 19th century and cannot be derived from Newtonian mechanics. Students accept the FLT easily, because it seems so reasonable from everyday experience, including frequent discussions of energy gain and loss. Introducing and using the FLT by name in the mechanics course provides a solid foundation for the later study of thermodynamics, where the FLT will appear as something familiar rather than as a correction to an incomplete mechanical work–energy equation, or even as an entirely new concept.

Changing the approach to work and energy will require significant effort. The potential rewards are great, however, in breaking down some of the present isolation of mechanics from thermodynamics. Moreover, it is possible to draw significant dividends within the mechanics course from investments made in dealing more comprehensively with energy. During the energy portion of the course we are able to discuss examples of the FLT involving rotating objects, where we simply refer to the rotational kinetic energy as $\mathrm{KE}_{\mathrm{rot}}$, not yet being able to calculate it explicitly. For example, the CM and FLT equations for a cylinder rolling down an incline can be discussed at an early stage. This makes it possible later to draw upon this experience when introducing the kinetic energy of rotating objects, and the moment of inertia. The use of the CM equation and the FLT makes it feasible to attack a wide range of rotation problems after having merely introduced $\frac{1}{2} I\omega^2$ and rotational kinematics, leaving until later the more subtle concepts of torque and angular momentum. This gets over the hurdle so common to the topic of rotational dynamics of having to introduce a large number of new concepts all at once, in order to be able to do anything new and interesting. I acknowledge a debt to James H. Smith for pointing out the advantages of energy techniques in introducing rotational mechanics.

VII. CLOSING REMARKS

My own concern with these issues began in 1971 when Lynell Cannell and I became very confused about missing thermal energy terms in what we now recognize as the CM equation. This was in the context of her starting to write a computer-based lesson on work and energy, and we found the requirements for extra clarity in the computer-based education medium a great stimulus to sorting things out. Our colleague James Smith came to the rescue with the basic explanations elaborated upon in this article. Over the ensuing years a great deal of experience has been gained in reorienting our teaching of work and energy. Some of this experience is reflected in a computer-based lesson on work, energy, and the CM equation, and in its accompanying problem set. These and related concerns have also led to increased emphasis on freebody diagrams throughout our computer-based course.[9]

It must be admitted that our attempts to treat work and energy comprehensively have not been completely successful. Students do find the topic difficult. It is observed that the first law of thermodynamics leads to few conceptual difficulties, but the CM equation strikes the students as rather mysterious. Perhaps this is a reflection of the fact that the foundation of the CM equation, Newton's second law for a system of particles ($\Sigma\, \mathbf{F}_{\mathrm{ext}} = M\mathbf{a}_{\mathrm{CM}}$), *is* in fact rather mysterious, since it relates the motion of the center of mass to forces applied at points which may be far from the center of mass.

The difficulties students have with the new presentation may be understood in part as being no different from comparable difficulties with other parts of the mechanics course, such as rotational motion and angular momentum, but partly the difficulty lies with the newness of the approach. The standard textbooks have not handled the topic in this way. Moreover, as was mentioned earlier, examples and problems which bring out the issues have not been included in textbooks and are not part of every physics teacher's set of teaching methods. Senior faculty members and graduate teaching assistants, having studied the material in a very different way when they were undergraduates, need assistance in presenting things the new way. In short, there is a lack of instructional infrastructure to support the revised treatment of work and energy. In order to remedy this situation we have not only used computer-based materials and (in preliminary form) a new textbook[10] but have also held special seminars for staff.

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[1]H. Erlichson, Am. J. Phys. **45**, 769 (1977).
[2]C. Penchina, Am. J. Phys. **46**, 295 (1978).
[3]B. A. Sherwood, C. Bennett, C. Tenczar, and J. Mitchell, *Proceedings of the Conference on Computers in the Undergraduate Curricula* (Dartmouth College, Hanover, NH, 1971).
[4]S. Smith and B. A. Sherwood, Science **192**, 344 (1976). Reprinted in *Electronics: The Continuing Revolution*, edited by P. H. Abelson and A. L. Hammond (American Association for the Advancement of Science, Washington, DC, 1977).
[5]D. Kane and B. Sherwood, Comp. and Educ. **4**, 15 (1980).
[6]L. M. Jones, D. Kane, B. A. Sherwood, and R. A. Avner, Am. J. Phys. **51**, 533 (1983).
[7]D. Halliday and R. Resnick, *Fundamentals of Physics*, 2nd ed. (Wiley, New York, 1981).
[8]B. A. Sherwood (unpublished).
[9]B. A. Sherwood, Am. J. Phys. **39**, 1199 (1971).
[10]B. A. Sherwood, *Notes on Classical Mechanics* (Stipes, Champaign, IL, 1982).

Timing the flight of the projectile in the classical ballistic pendulum experiment

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An apparatus has been designed and constructed to measure the time of flight of the projectile fired by the Blackwood pendulum apparatus. Microphones mounted on the pendulum base and on a metal target plate yield signals, which after amplification and shaping by custom designed circuitry, start and stop a commercial digital timer. The purpose, use, and design of these accessories are described.

For many years, students in the algebra-based introductory physics sequence at Iowa State have performed a ballistic pendulum experiment with traditional apparatus.[1] It has been our experience that while this experiment illustrates quite well several important principles and is enjoyed by our students, it is a relatively brief exercise. Recently, as part of the development of a new laboratory for our calculus-based sequence we have redesigned some aspects of this experiment to increase the diversity of measurements that the students make. Specifically, we have designed an apparatus which provides a signal to start a commercial digital timer when the spring gun is fired and a second signal to stop the timer when the ball strikes the floor. Students use this equipment to measure the time of flight of the ball. From this, they calculate the ball's horizontal velocity, a result which is then compared with that obtained from the pendulum measurements. In the traditional procedure, the time of flight of the ball is assumed to be the same as that for a ball simply dropped from the same elevation as that of the gun. Students would normally measure the elevation of the gun and calculate the time of flight rather than measure it directly. We have also designed a simple fixture which provides a signal to start a timer when the ball is dropped from rest. This additional apparatus enables the student to time the free fall of the ball from any desired height. Thus, in addition to permitting a more direct determination of the velocity of the projectile, these accessories also make possible a direct test of the assumption that the time of flight of a projectile over a level surface is independent of its initial horizontal velocity.

Unlike previously reported apparatus to measure the time of flight of a projectile,[2–4] the accessories that we have designed make use of microphone cartridges. Signals from the microphones, after amplification and shaping, provide start and stop signals for a commercial digital timer.[5] We make use of low-cost replacement cartridges for dynamic microphones,[6] one of which is mounted with epoxy to the underside of the metal base of the pendulum apparatus, directly under the gun. A second is epoxied to a 1.9-mm-thick (14 gauge) galvanized steel plate which is placed on the floor at the location where the ball is expected to strike. The microphone is mounted near one corner of the plate and housed within a rugged steel electronics box for protection from the projectile. A phono jack is mounted on both

