Blau and Halfpap posed the question in the American Journal of Physics of how to interpret refraction (Snell's law; index of refraction) and the (apparent) slower speed of light in glass in terms of quantum mechanics. The following response by Bruce Sherwood was published in the American Journal of Physics 64, 840-842 (1996).

Answer to Question #21. ["Snell's law in quantum mechanics," Steve Blau and Brad Halfpap, Am. J. Phys. 63(7), 583 (1995)]

The question of how to interpret Snell's law and the index of refraction from the point of view of photons and quantum mechanics can usefully be recast as a question of how to interpret these concepts from a microscopic point of view, whether quantum-mechanical or (semi-)classical. Feynman has an excellent microscopic analysis of the index of refraction in his Chapter 31 on "The Origin of the Refractive Index." [Footnote 1] He points out that, "so far as problems involving light are concerned, the electrons (in atoms) behave as though they were held by springs" (p. 31-4). One can glimpse how this is possible by approximating the electron cloud in hydrogen with a uniform-density sphere of radius R. If the proton is displaced a distance s from the center of this sphere, there is a restoring force on the proton due to the electric field contributed by that portion of the sphere that is inside the radius *s:*

$$F = q_{\text{proton}} E_{\text{electron sphere}} = e\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{e}{\frac{4\pi}{3}R^3}\right)\frac{4\pi}{3}s^3\right]\frac{1}{s^2} = \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^3}\right)s$$

This is also the force exerted on the electron cloud by the proton, and it is proportional to s, just like a spring force. Of course quantum mechanics is required to predict the actual electron charge distribution, but many of the electric consequences of that charge distribution can be analyzed classically.

In a microscopic but otherwise classical analysis, the electric field in electromagnetic radiation accelerates electrons held by springs in the atoms of a piece of glass, and these accelerated electrons re-radiate in all directions. The observed light is the superposition of the electric (and magnetic) fields of the incoming light and the re-radiation. Full quantitative analysis from a microscopic point of view requires a kind of self-consistent calculation, because the re-radiation from accelerated electrons contributes to the net electric field driving the electrons. Feynman deals with the low-density limit in which re-radiation of re-radiation is negligible, but this is adequate to understand the essential aspects of the phenomena.

In the backward direction we normally call the re-radiation "reflection," but this labeling obscures the fact that this is new light radiated by all the atoms in the glass, not old light that has magically "bounced off" the front surface due to some unknown mechanism. The microscopic analysis of "reflection" is exactly the same as the analysis of x-ray diffraction, but because the inter-atomic spacing is small compared to the wavelength of visible light, the "reflected" light has just one, zeroth-order interference maximum in the "reflection" direction (n = 0 in the Bragg "reflection" condition).

In the forward direction we speak of "refraction," and we say that "the speed of light is slower in the glass," but in fact, the speed of light does not change in the material. Rather, Feynman shows how the superposition of the incoming light, traveling at speed *c*, and the light re-radiated by the atomic electrons, traveling at speed *c*, shifts the phase of the radiation in the air downstream of the glass in the same way that would occur if the light were to go slower than *c* in the glass, with a shorter wavelength and an index of refraction greater than one for frequencies below the natural frequency of the oscillators (otherwise the phase shift corresponds to a speed greater than *c* in the material, with index of refraction less than one). At a fundamental level this phase velocity, greater or less than *c*, is of no particular physical significance, because it only applies to the overly-simplified case of single-frequency sinusoidal radiation permeating all space, and such radiation cannot carry a meaningful signal.

To see what happens to a meaningful signal, consider incoming radiation in the form of a sine wave that starts suddenly. Detecting the leading edge of this sine wave provides real information. Suppose the electric field in the first half cycle is in the $+y$ direction. An electron is initially driven downward by the incoming electric field, and the accelerated electron radiates in (nearly) all directions. In the forward direction the re-radiated field at far distances is proportional to $-qa_y = +ea_y$ (with a_y negative), so that the contribution to the net downstream field is in the $-y$ direction. Therefore, at a downstream observation point, the net field during the beginning of the cycle is reduced slightly from what it would be in the absence of the charge on a spring, and the rise time is slower.

The details of the net wave shape can be studied by numerical integration of the motion of the electron on the spring under the influence of the sudden-onset incoming sine wave, to obtain the acceleration of the electron as a function of time to use in evaluating the charge's $-qa_y$ contribution to the net field. One finds by numerical computation that the effect of a single oscillator is to make the initial maximum of the net field at the observation point occur slightly late, as though the speed of light were less than *c*.

However, the apparent "slower speed" is the result of the superposition of two radiative electric fields, the incoming radiation and the re-radiation, both of which travel at the normal speed of light *c*. If taken too seriously, it is a violation of the superposition principle to say that the speed of light is affected by the presence of matter. The incoming radiation was produced by some accelerated charges, and the field that those charges produced is unaffected by the presence of other charges anywhere in the universe, and this field propagates at speed *c*. In particular, incoming radiation passes through glass unchanged, but downstream we observe the superposition of this unchanged radiation with re-radiation from the accelerated electrons in the glass. The leading edge of radiation may travel at a speed smaller than *c*, but only through the superposition of the contributions of accelerated charges that make radiative fields that propagate at speed *c*.

Another relevant computation is to re-do numerically Feynman's analytical calculation for a very thin glass plate, but with a sudden-onset sine wave instead of a continuous sine wave. For a driving frequency below the natural frequency of the oscillators, one finds a slight delay in the first maximum, and one also finds in agreement with Feynman's calculation a phase delay after the steady state is attained. For a driving frequency above the natural frequency of the oscillators, there is hardly any delay in the first maximum, and there is a phase advance after the steady state is attained, as discussed by Feynman in Section 31-4 on "Dispersion."

It is enormously convenient to describe refraction by saying "the speed of visible light is smaller in glass." It would be extremely difficult from the microscopic viewpoint to calculate the index of refraction for a dense material such as glass, due to having to take into account re-radiation of re-radiation in a self-consistent way, using the correct form of the retarded fields of nearby accelerated electrons (Feynman's calculation not only neglects re-radiation of re-radiation but requires only the far-field approximation). The index of refraction lumps all of this complexity into one convenient number, and one convenient metaphor.

The mathematical complexity of the microscopic analysis is prohibitive for most quantitative work, but it complements the macroscopic picture by providing a deep sense of mechanism and by permitting a unified microscopic analysis of reflection, refraction, x-ray diffraction, and even thin-film interference. This is analogous to the insight that kinetic theory adds to thermodynamics, or that circuit analysis in terms of surface charge [Footnote 2] adds to the Kirchhoff loop and node rules.

The original question asked about Snell's law from the point of view of photons. The main issue isn't really photons, but microscopic versus macroscopic analyses. The passage to quantum mechanics introduces still more mathematical complexity but doesn't change the main point. The reflected and refracted light consists of the (quantum) interference of incoming photons with photons re-emitted by atoms in the glass. The fundamental speed of light is unaffected.

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1. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading MA, 1963), Chapter 31.

2. R. W. Chabay and B. A. Sherwood, *Matter & Interactions: Electric & Magnetic Interactions* (Wiley, New York, 2002), Chapter 18. Also see Chapter 23 on the classical interaction of light and matter.