Sparks: An example of competing models

Bruce Sherwood & Ruth Chabay

Department of Physics

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NC State University

Charge transfer through air

- Model A: Electrons jump through the air from one electrode to the other
  - The mean free path is $d \approx 5 \times 10^{-7}$ m
  - So model A is ruled out
- Model B: If the air were ionized, ions and free electrons could drift through the air
  - A displacement of even one mean free path can correspond to a large transfer of charge:

$$neA\Delta x = \left(\frac{6 \times 10^{23} \text{ ions}}{22.4 \times 10^{3} \text{ cm}^{3}} \right) \left(1.6 \times 10^{-19} \text{ C}\right) \left(0.1 \text{ cm}\right)^{2} \left(5 \times 10^{-5} \text{ cm}\right) \approx 2\ \mu\text{C}$$

Why the short duration? Why the emitted light?

- Charge transfer reduces the charges on the electrodes, reducing the electric field between the electrodes
  - Observe that $E_{\text{critical}} \approx 3\times10^{6}\,\frac{\text{N}}{\text{C}}$
  - When $E < E_{\text{critical}}$, the spark extinguishes
- Recombination of ions and free electrons
  - Light emitted as the neutral atoms drop toward the electronic ground state

Model 1 for ionizing the air: Direct ionization

- To produce a spark in air, apply an electric field large enough to rip electrons out of the air molecules
- Estimate the critical field strength:

$$E_{\text{critical}} \approx \frac{1}{4\pi\varepsilon_0}\frac{e}{r_{\text{atom}}^2} \approx \left(9\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)}{\left(10^{-10}\ \text{m}\right)^2} \approx 10^{11}\ \frac{\text{N}}{\text{C}}$$

- Observed: $E_{\text{critical}} \approx 3\times10^6\ \frac{\text{N}}{\text{C}}$
- We're off by five orders of magnitude; minor tweaking of the model won't help

Model 2 for ionizing the air: A chain reaction

- Suppose there is a free electron somewhere, somehow
- Suppose field is big enough that this electron gains enough kinetic energy in one mean free path to ionize an air molecule
- Now there are 2 free electrons, then 4, 8, 16, 32…a chain reaction
- The ionized air is now a conductor; a spark is formed

Where can we get a free electron?

- Cosmic rays (mainly high-energy protons) collide with nuclei in the upper atmosphere and produce many particles
- Only the muons have long enough lifetimes (thanks to time dilation) and ranges to reach the Earth’s surface
- The charged muons leave a trail of ionization in the air
- Natural radioactivity also contributes to there being free electrons in the air

Estimating the critical field strength

- Assume that an electron loses almost all its kinetic energy in a collision and starts the next acceleration nearly from rest:

$$K_i \approx 0$$

$$K_f \approx \Delta K = eEd \approx |U_{\mathrm{el}}| = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\mathrm{atom}}}$$

$$E_{\mathrm{critical}} \approx \frac{|U_{\mathrm{el}}|}{ed} \approx \frac{1}{4\pi\varepsilon_0}\frac{e}{r_{\mathrm{atom}}d}$$

$$E_{\mathrm{critical}} \approx \left(9\times10^9\,\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \mathrm{C}\right)}{\left(10^{-10}\ \mathrm{m}\right)\left(5\times10^{-7}\ \mathrm{m}\right)} \approx 30\times10^6\,\frac{\mathrm{N}}{\mathrm{C}}$$

Good or bad agreement?

- The prediction of model 2 is off by one order of magnitude; is that good or bad agreement?
- We have not taken into account important statistical aspects: some electrons will travel much farther than one mean free path before colliding, nor is it necessary that *every* electron cause ionization.
- Tentatively conclude that this model is viable.
- But a really good model would predict/explain more than just the original goals of the model.

Additional predictions of models

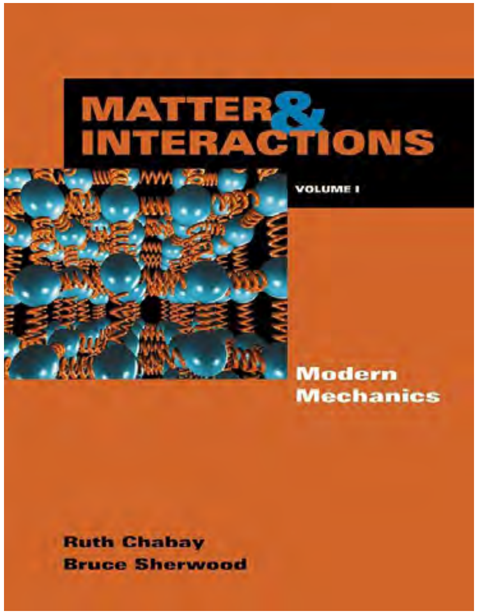
- How should E_{critical} change if we double the air density?
- What do the two models predict?

Additional predictions of models

- How should $E_{critical}$ change if we double the air density?
  - Model 1 (direct ionization): no dependence
  - Model 2 (chain reaction): half the mean free path, so need twice the field strength for *eEd* to equal the ionization energy
- Observe: double the density, double $E_{critical}$
- This is additional strong evidence for the validity of model 2

Pedagogical value of this case study

- Example of competing models that are accessible at the introductory level
- Intuitively appealing models can be ruled out on quantitative grounds (no leaping, no direct ionization by the field)
- Example of long chain of reasoning, each link accessible at the introductory level
- Example of a model explaining more than it was designed to do



Matter & Interactions I:
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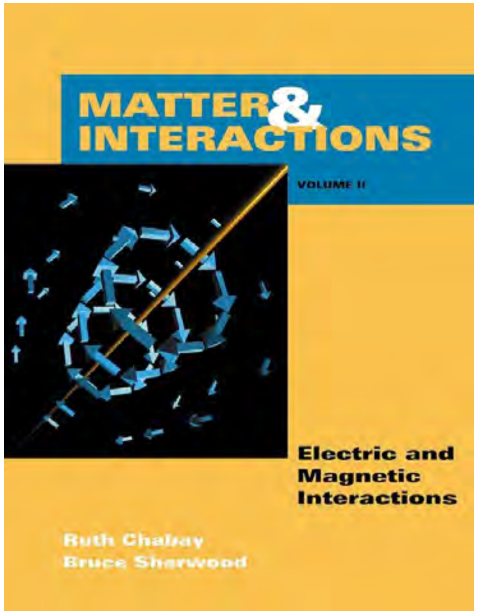

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Ruth Chabay & Bruce Sherwood
John Wiley & Sons, 2002

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