

Free-Body Diagrams (a PLATO Lesson)

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A lesson has been written for the PLATO computer-based education system which tutors students in a systematic approach to dynamics problems involving free-body diagrams. The lesson is described in detail and illustrated with photographs of the student's graphical display screen.

College students in the introductory mechanics course typically have serious difficulties with dynamics problems involving free-body diagrams. The principal difficulty is the inability to attack problems in a systematic way. Specific difficulties include failure to isolate a particular subsystem, unsuccessful attempts to use pseudoforces, and confusion of geometrical constraints with dynamic equations of motion. Sign errors plague the student.

In a typical class with 20 students even a good teacher can do little to remedy this situation. Every year students bring to the instructor's office hours the same kinds of confusion. Each student needs individual attention and a lot of practice. This requires a patient tutor.

In an attempt to offer effective personal attention to large numbers of students a lesson has been written in the TUTOR language for the PLATO computer-based education system.¹ Although rather flexible, the lesson does force the student to attack problems in a systematic way. The student must begin by choosing a subsystem. He names *objects* which can exert forces on this subsystem, which prevents him from introducing pseudoforces such as "centrifugal force." A sharp distinction is made between equations of motion and geometrical constraints. Sign errors are pointed out.

Figure 1 lists the three available problems and a summary of the steps to be followed. (In problems 1 and 2 the student can elect whether or not to consider friction.) The student must first choose a subsystem, next find the external forces, and then write the equations of motion. He must obtain the equations of motion for two different subsystems (in problems 1 and 2) before writing the constraint equations. When he has the equations of motion and the constraint equations, he is told to solve for the unknown quantities with pencil and paper. He is then asked for his results.

For each step, the student specifies in words what he wants to do: "Choose a subsystem" or "Choose m_1 ." "Find the forces." "Define coordinate axes." "Write equations of motion." "Write constraints." (Many variants of all these forms are permitted.) The lesson gives the student

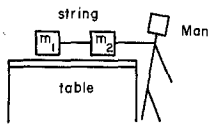
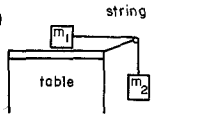
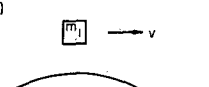

<p>The student can choose any of these problems:</p> <p>1)  Man</p> <p>Find tension, acceleration (table has friction)</p> <p>2)  string</p> <p>Find tension, acceleration (table has friction)</p> <p>3)  v</p> <p>earth</p> <p>Find v for circular orbit (force is GMm_1/r^2)</p>	<p>He must attack each problem in the following way:</p> <p>A) Choose subsystem $m_1, m_2,$ or m_1+m_2 (In prob.3, only m_1.)</p> <p>B) Find the external forces acting on the subsystem.</p> <p>C) Define coordinate axes.</p> <p>D) Write equations of motion.</p> <p>E) Repeat above steps for another subsystem.</p> <p>F) Write constraint equations</p> <p>Problem 1: $a_{1x} = a_{2x},$ $a_{1y} = 0, a_{2y} = 0.$</p> <p>Problem 2: $a_{1x} = -a_{2y},$ $a_{1y} = 0, a_{2x} = 0.$</p> <p>Problem 3: $a_{1x} = 0,$ $a_{1y} = -v^2/r.$</p> <p>(For constraints, the student must use axes defined by the lesson as .)</p> <p>G) Solve equations for unknowns.</p>
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FIG. 1. Problems 1, 2, and 3 and method of solution.

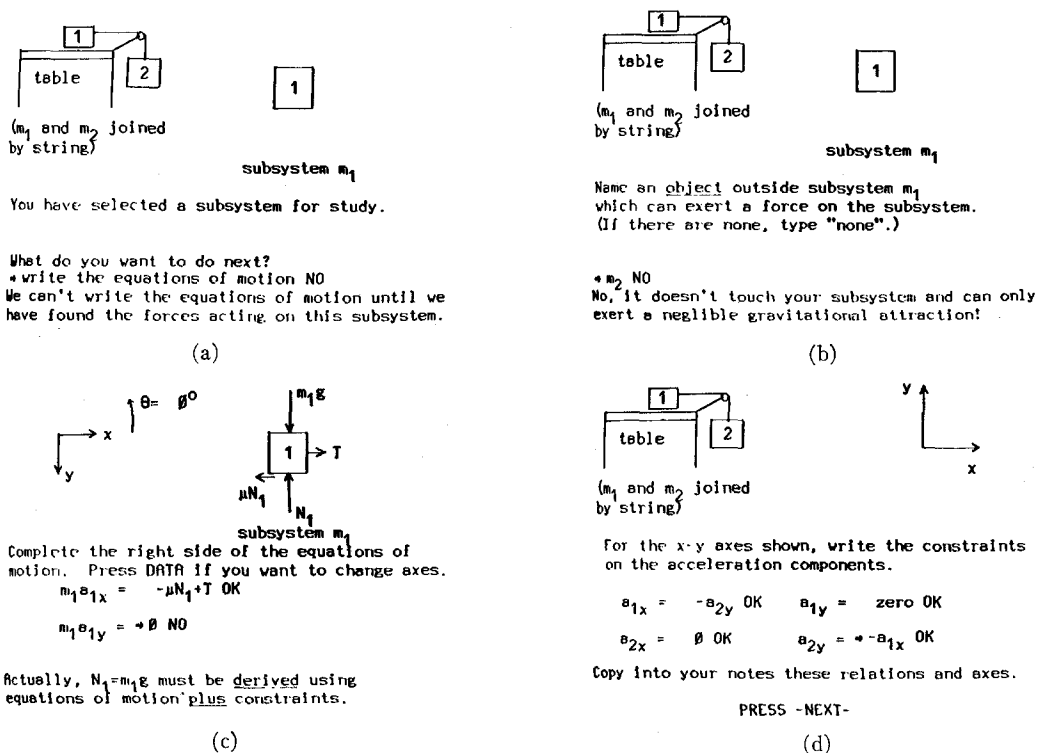


FIG. 2. Major steps in the solution of problem 1. (Photographs of the student's display screen.) (a) The student must proceed in an orderly way. (b) Careful distinction between contact forces and action at a distance. (c) Distinction between equations of motion and constraints. (d) Constraint equations (geometry).

an appropriate comment if he tries to perform some step out of order, such as asking to find the forces before choosing a subsystem. [See Fig. 2(a), which like the succeeding figures is an actual photograph of the student's graphical display screen.] If he asks prematurely to "solve the problem," the lesson tells him, "You don't yet have enough equations to solve the problem fully." At any time the student can refer to a "HELP" page which outlines the major steps to be followed.

ADDITIONAL DESCRIPTION OF THE VARIOUS STEPS

Forces: The student must specify an object which can exert a force on the subsystem. [See Figs. 2(b) and 3(a)]. When he does so, the labeled vector is immediately added to the picture of the subsystem on the student's display screen. If the object (other than the earth) does not touch the subsystem, he is told, "That object can only exert a negligible gravitational attraction." Responses such as "gravity" or "friction" bring the reply, "That's not an object. What object exerts

that force?" The object "air" yields the reply, "We'll neglect air resistance." In problem 3 the words "centrifugal" or "centripetal" cause a reply, "That's not an object." Note that this emphasis on objects prevents the student from introducing pseudoforces. A "HELP" page emphasizes that only contact forces are important except for the "action at a distance" of the massive Earth.

Axes: If the student does not ask to define coordinate axes, standard axes like those in Fig. 3(b) are set up. He can redefine the axes at any time [the phase "Press DATA" in Fig. 3(b) refers to a key labeled "DATA" on the student console keyboard]. After going to the axes definition page, he can specify a rotation about the z axis by giving a value for θ in degrees. He can then change the sign of the y axis if desired.

Equations of motion: Consider subsystem m_1 of problem 2. [See Fig. 2(c)]. The student must complete the equations

$$m_1 a_{1x} = ?$$

$$m_1 a_{1y} = ?$$

(m_1 and m_2 connected by a string; man touches m_2)

See the force displayed above?
Now name another object which can exert a force.
(Or type "none" if there are no more.)
*gravity NO
That's not an object! What object exerts that force?

Complete the right side of the equations of motion. Press DATA if you want to change axes.
 $m_1 a_{1x} + m_2 a_{2x} = T - \mu(N_1 + N_2)$ OK
 $m_1 a_{1y} + m_2 a_{2y} = -N_1 - N_2 - (m_1 + m_2)g$ OK

Copy the equation and axes into your notebook.

PRESS -NEXT-

Complete the right side of the equations of motion. Press DATA if you want to change axes.
 $m_1 a_{1x} = 0$ OK
 $m_1 a_{1y} = + m_1 (GM/r^2)$ NO

You have your signs backwards.

For the x-y axes shown, write the constraints on the acceleration components.
 $a_{1x} = 0$ OK $a_{1y} = +v^2/r$ NO

You have your signs backwards.

FIG. 3. Major steps in problems 2 and 3. (Photographs of the student's display screen.) (a) Example of compound system. (b) Equations can be written in any algebraically equivalent form. (c) Checking for sign errors in equations of motion. (d) Sign error in constraint equation.

For the axes shown, the correct answers are

$$m_1 a_{1x} = T - \mu N_1$$

and

$$m_1 a_{1y} = m_1 g - N_1.$$

Equivalent algebraic expressions are permitted, and implied multiplication (without a multiplication sign) is allowed. For example,

$$m_1 a_{1x} = -3\mu(N_1/3) + 5T - 4T$$

is considered correct. The lesson responds specifically to certain "wrong" answers (or their algebraic equivalents):

$m_1 a_{1x} = \mu N_1 - T$ "Your signs are backwards."

$m_1 a_{1x} = T$ "You forgot the frictional force."

$m_1 a_{1x} = T - \mu m_1 g$ " $N_1 = m_1 g$ must be derived from the equations of motion plus constraints."
or
 $m_1 a_{1y} = 0$

See Figs. 3(b) and 3(c) for other examples. A "HELP" page explains the simple techniques for typing capital letters, " μ ," superscripts, and subscripts. The student is told that he may write either $m_2 g$ or $m_2 g$.

Constraints: Specific "HELP" pages for each problem explain briefly the geometrical nature of the constraints. The problems include examples of $a_y = 0$, $a_{1x} = a_{2x}$, and $a_r = -v^2/r$. The student is asked to complete the equations:

$$a_{1x} = ?, \quad a_{1y} = ?,$$

$$a_{2x} = ?, \quad a_{2y} = ?.$$

If the sign is wrong, he is told so. Other wrong answers merely generate the reply "Press HELP for a discussion of constraints." [See Figs. 2(d) and 3(d).] For the constraint equations the student must use the standard axes shown.

Solve: When the student successfully writes the equations of motion, and when he correctly writes the constraint equations, he is told to copy his equations into his notebook. After all the equations

have been generated, he is told to do the algebra with pencil and paper to solve for the unknown quantities of interest. He is asked for his results. Because the lesson is not meant to teach algebra, the student is allowed to see the correct results by pressing the "HELP" key.

REMARKS

The lesson has been tried by volunteer students. This lesson is one part of a computer-based mechanics course under development.²

Many of the typical homework problems involving free-body diagrams fit into the framework of this lesson. The structure of the lesson is such that it is easy to add additional problems of this type to the present selection.³

Student responses recorded by the computer and informal conversations with students have been instrumental in developing the lesson to its present form. This development has been mainly in the direction of making the lesson less rigid. Because the lesson gives a lot of help, it would be useful

to follow up this lesson with a computer quiz which would offer minimum help to the student. Performance on the quiz could be used to determine whether some of the lesson should be repeated by the student.

It appears that this kind of computer-aided study can make a lecture or classroom discussion more valuable. After the students have completed their computer lesson, the lecturer or discussion leader can count on a reasonably high minimum level of understanding and a common active vocabulary. These factors permit discussion of finer points and shortcuts. For example, the acceleration in problem 1 follows immediately from considering both masses as the subsystem of interest. Students poorly prepared by conventional study do not fully appreciate this shortcut. While studying the computer lesson a student is forced to be much more active than when studying on his own. As a result, he gets a clear idea of what a subsystem is and can appreciate the advantage of simplifying the equations of motion by a judicious choice of subsystem.

¹ D. L. Bitzer, R. Blomme, B. Sherwood, and P. Tenczar, "The PLATO System and Science Education," in *Proceedings of a Conference on Computers in Undergraduate Science Education, Illinois Institute of Technology, Commission on College Physics, College Park, Md., August 1970* (to be published in 1971).

² B. Sherwood, C. Bennett, J. Mitchell, and C. Tenczar, "Experience with a PLATO Mechanics Course," in *Proceedings of a Conference on Computers in the Undergraduate Curricula, Dartmouth College, Hanover, New Hampshire, June 1971*.

³ Recent additions include a block sliding down an incline with friction and four problems involving two-dimensional rotation. In the rotational problems the student must write, in addition to the force equations and acceleration constraints, a torque equation and a constraint on the angular acceleration. The addition of the incline problem forced a change in the treatment of the constraints: The constraints are now written in terms of the magnitude " a " of the acceleration of m_1 .